A Lean 4 Formalization of the Strong Perfect Graph Theorem

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Abstract

We report a complete formalization, in Lean 4 with Mathlib, of the Strong Perfect Graph Theorem of Chudnovsky, Robertson, Seymour and Thomas: a graph is perfect if and only if it is Berge. A graph is *perfect* if, in every induced subgraph, the chromatic number equals the size of the largest clique; it is *Berge* if neither it nor its complement contains an induced odd cycle of length at least five. The Lean proof follows the published paper statement by statement, proves every result the paper cites from prior work, and depends on no axiom beyond Lean's kernel. Three statements used by the proof are false as printed; we give their corrected forms and, for Theorem 9.3, a machine-checked counterexample to the printed statement. Smaller repairs to individual proof steps are collected in an appendix.

Lean formalization:
`https://beyondthelibrary.github.io/formal_archive/p/StrongPerfectGraphTheorem/`

1 Introduction

A graph G is *perfect* if for every induced subgraph H of G the chromatic number of H equals the size of the largest clique of H. A *hole* is an induced cycle of length at least four, an *antihole* is an induced subgraph whose complement is a hole, and G is *Berge* if every hole and antihole of G has even length. In 1961 Berge [1] proposed two conjectures: that the complement of every perfect graph is perfect, and that a graph is perfect if and only if it is Berge. The first, the *weak* perfect graph conjecture, was proved by Lovász in 1972 [11]. The second, the *strong* perfect graph conjecture, remained open for forty years. It was proved by Chudnovsky, Robertson, Seymour and Thomas, and the result is now the Strong Perfect Graph Theorem:

Theorem (1.2 of [3])**.** *A graph is perfect if and only if it is Berge.*

The theorem matters for several reasons. Perfect graphs are the class on which the chromatic number, the clique number, and the Shannon capacity coincide and can be computed in polynomial time by the ellipsoid method [9]; the theorem replaces the definition of perfection, which quantifies over all induced subgraphs, by a purely structural condition on induced cycles. The proof does more than settle the conjecture: it establishes the structure theorem (Theorem 1.3 of [3], conjectured in this form by Conforti, Cornuéjols and Vušković) that every Berge graph either lies in one of a few basic classes or admits one of a few decompositions, which is the form in which the result has since been used.

The proof was completed in 2002. The manuscript is dated June 20, 2002, and was posted to the arXiv on December 4, 2002 as `math/0212070` (AIM preprint 2002-19, 150 pages) [2]. The paper was then revised; the published version carries the date line "June 20, 2002; revised July 19, 2005" and appeared in Annals of Mathematics, volume 164 (2006), pages 51–229 [3].

The revision changed the statements of 4.6, 5.7, 8.5, 11.3, 12.1 (outcome 1), 13.6, 15.5, 21.2 and 22.5, and the definition of a *tail*. The proof was awarded the 2009 Fulkerson Prize.

We formalized the published paper in Lean 4 [7], version 4.30.0-rc2, on top of Mathlib [12] at commit `2f9133a` of 21 April 2026, following the published text in every case where the two versions differ. The result is about 350,000 lines in 1,209 modules whose root declaration, `thm_1_2`, is the Strong Perfect Graph Theorem stated for an arbitrary finite simple graph. It is accepted by the Lean kernel and depends on exactly Lean's three foundational axioms, `propext`, `Classical.choice` and `Quot.sound`; the formalization contains no `sorry`, no `axiom` and no `opaque` declaration. In particular nothing the paper takes from the literature is assumed. The paper cites eight results without proof: Lovász's theorem 1.1, the Roussel–Rubio lemma 2.1 [13], and the six facts used on page 4, namely that a minimum imperfect graph admits no proper 2-join [6], no proper homogeneous pair [5] and no star cutset [4], that basic graphs are perfect, that perfect graphs are Berge, and Lovász's replication lemma [10]. All eight are proved in Lean, together with Dirac's theorem on subdivisions of K_4 [8], which the printed proof of 5.3 uses without citation; 2.1 is proved through a stronger parity form and converted to the printed statement.

The formalization succeeded with a few fixes. Three numbered statements used by the proof are false as printed and had to be corrected before they could be proved; Section 2 gives them, with the counterexample to Theorem 9.3. One item of the summary Statement 1.8 and a number of individual proof sentences had to be repaired; Appendix A lists these in tabular form. None of these fixes touches the main theorem or the structure theorem.

2 Major issues: statements that are false as printed

Three numbered statements that the proof uses are false as printed. In each case the Lean states a corrected version, the correction is the one the paper's own proof or its own later use supplies, and every use of the statement elsewhere in the paper goes through with the corrected form. We list them from the most to the least consequential. For each statement we give the statement exactly as printed (red bar) and the corrected statement in the paper's wording with the change marked (blue bar; ~~struck~~ text is removed, underlined text is added); the corrected statement is the one proved in Lean. Page numbers are printed page numbers.

2.1 Theorem 9.3, outcome 4

9.3 as printed (p. 48)

9.3 Let (P_1, P_2, Q_1, Q_2) be a knot in a Berge graph G, inducing K. Assume that there is no appearance in G or in $\overline{G}$ of any K_4-enlargement, and there is no overshadowed appearance of K_4 in G or in $\overline{G}$. Let F be a connected subset of $V(G) \setminus V(K)$, such that its set of attachments in K is not local. Then either:

1. there is a vertex in F such that its neighbour set in K resolves the knot, or
2. (up to symmetry) there is a path R in F with ends r_1, r_2 such that r_1, a_1 have the same neighbours in $V(P_2) \cup V(Q_1) \cup V(Q_2)$, and there are no edges between $R \setminus r_1$ and $V(P_2) \cup V(Q_1) \cup V(Q_2)$, and r_2 has a neighbour in $P_1 \setminus a_1$, and there are no edges between $R \setminus r_2$ and $P_1 \setminus a_1$, or
3. (up to symmetry) there is an odd path R in F with ends r_1, r_2 such that r_1, a_1 have the same neighbours in $V(P_2) \cup V(Q_1) \cup V(Q_2)$, and r_2, b_1 have the same neighbours in $V(P_2) \cup V(Q_1) \cup V(Q_2)$, and there are no edges between $V(R^*)$ and $V(P_2) \cup V(Q_1) \cup V(Q_2)$, and no edges between R and P_1 except possibly $r_1 a_1$ and $r_2 b_1$, or

4. there is a vertex $f \in F$ such that (up to symmetry) f, x_1 have the same neighbours in $V(P_1) \cup V(P_2) \cup V(Q_2)$ and f is not adjacent to y_1.

What is wrong. The clause "f is not adjacent to y_1" in outcome 4 is false. It holds when Q_1 has length 1 (case (1) of the printed proof), but in case (2), where Q_1 may be long, the argument only produces a non-neighbour of f on Q_1 other than x_1, and that non-neighbour need not be y_1.

The counterexample. The formalization contains an eleven-vertex graph, verified by the Lean kernel, that satisfies every hypothesis of 9.3 and none of the four printed outcomes (`Workspace.Counterexamples.Thm93.printed_nine_three_false`). Let H be the bipartite subdivision of K_4 with branch vertices $0, 1, 2, 3$, square 0-1-2-3-0, and diagonal branches 0-4-5-6-2 and 1-7-3. Label its ten edges as in Table 1. Let G_0 be the complement of the line graph $L(H)$ on these ten vertices; then

$$P_1 = a_1 b_1, \quad P_2 = a_2 b_2, \quad Q_1 = x_1 z_1 z_2 y_1, \quad Q_2 = x_2 y_2$$

is a knot in G_0 inducing $K = V(G_0)$. Let G be obtained from G_0 by adding one vertex f adjacent to exactly $a_1, a_2, x_1, z_2, y_1, x_2, y_2$, that is, to everything in K except b_1, b_2, z_1; so f is a twin of x_1 in $\overline{G}$ within K. Take $F = \{f\}$. Figure 1 draws the graph.

Table 1: The ten edges of H and their labels as vertices of $G_0 = \overline{L(H)}$. In G_0 two labels are adjacent exactly when the corresponding edges of H are disjoint.

edge of H	01	12	23	30	04	45	56	62	17	73
label	b_1	a_2	a_1	b_2	x_1	z_1	z_2	y_1	x_2	y_2

The Lean proof checks each hypothesis of `thm_9_3` for this configuration: G is Berge (neither G nor $\overline{G}$ has an odd induced 2-regular subgraph); the four lists form a knot inducing K; $F \subseteq V(G) \setminus K$ is connected; the attachments of F are not local; G has no appearance of K_4 at all (a degree count on every vertex subset of size at least 8), so neither an overshadowed one nor a K_4-enlargement appearance; and every appearance of K_4 in $\overline{G}$ is degenerate and has no odd branch of length at least 3, so $\overline{G}$ has no overshadowed appearance and no K_4-enlargement appearance either. It then shows that all four outcomes fail:

1. the neighbour set of f in K does not resolve the knot, since the edge $b_1 z_1$ has neither end in it;

2, 3. the only path in F is the single vertex f, and f is non-adjacent to z_1 while each of a_1, b_1, a_2, b_2 is adjacent to z_1, so f cannot share its neighbours on the other side of the knot with any a_i or b_i;

4. with $(x, y) = (x_1, y_1)$, f and x_1 do have the same neighbours in $V(P_1) \cup V(P_2) \cup V(Q_2)$, but f *is* adjacent to y_1; the three other symmetric labellings fail at a_1, b_2 and b_1 respectively.

Eleven vertices is the minimum: a knot needs $|K| \geq 8$, outcome 4 only bites when Q_1 is long, 9.1 forces its length to be even, so $|V(Q_1)| \geq 4$ and $|K| \geq 10$, and F needs one more vertex.

Correctness of the counterexample. The theorem `printed_nine_three_false` is checked by the Lean kernel, with no `sorry` and no axiom beyond `propext`, `Classical.choice` and `Quot.sound`. It states that every hypothesis of 9.3 holds for this configuration, that all four printed outcomes fail, and, through its companion `repaired_conclusion11`, that the corrected outcome 4 holds.

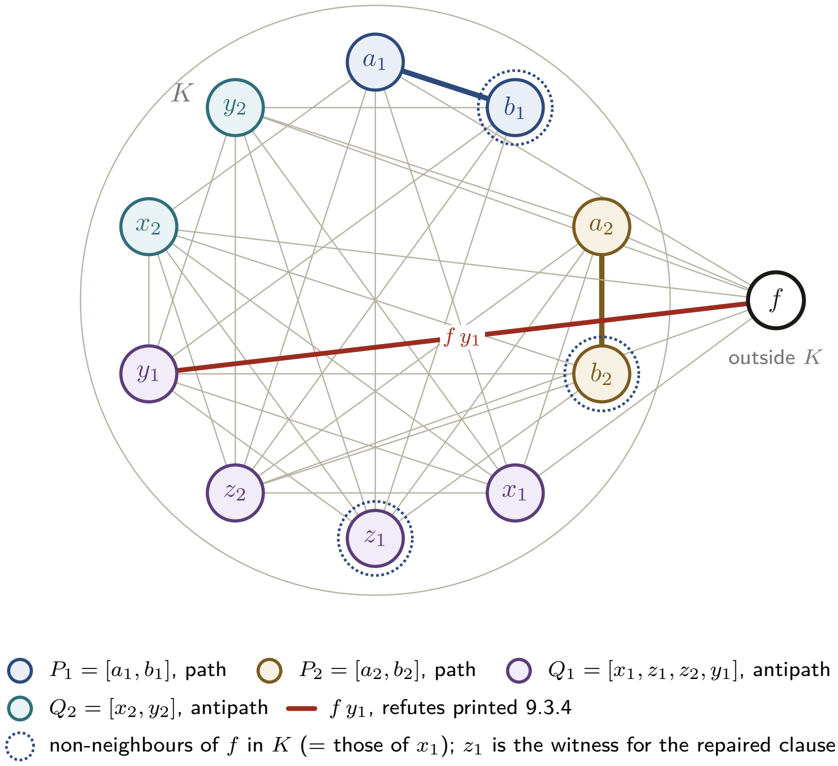

Figure 1: The eleven-vertex counterexample to Theorem 9.3 as printed. The ten vertices of the knot K sit on a circle in list order, coloured by the list they belong to; f lies outside K. All thirty-six edges of G are drawn and no non-edge is drawn: the gaps between consecutive vertices of Q_1 and of Q_2 are the antipaths. f agrees with x_1 on $V(P_1) \cup V(P_2) \cup V(Q_2)$ but is adjacent to y_1 (red edge), so printed outcome 4 fails; the absent edge fz_1 is the witness $w = z_1$ for the repaired clause.

The fix. Only the last clause of outcome 4 changes. The printed clause implies the corrected one, and the two coincide when Q_1 has length 1, so case (1) of the printed proof is unchanged; case (2) proves the corrected clause. The counterexample satisfies the corrected outcome 4 with witness $w = z_1$ (`repaired_conclusion11`), so the repair is neither vacuous nor too generous. Both consumers of 9.3 in the paper, 9.4 and 9.5, discard this conjunct of outcome 4 entirely, so nothing downstream changes. The erratum is recorded in the formalization's errata file `formaliaxiv.json`, anchored to the printed sentence, and in the module docstring of `Thm_9_3.lean`.

> **9.3 corrected (outcomes 1–3 unchanged)**
>
> **9.3** Let (P_1, P_2, Q_1, Q_2) be a knot in a Berge graph G, inducing K. Assume that there is no appearance in G or in $\overline{G}$ of any K_4-enlargement, and there is no overshadowed appearance of K_4 in G or in $\overline{G}$. Let F be a connected subset of $V(G) \setminus V(K)$, such that its set of attachments in K is not local. Then either:
>
> 1–3. as printed, or
> 4. there is a vertex $f \in F$ such that (up to symmetry) f, x_1 have the same neighbours in $V(P_1) \cup V(P_2) \cup V(Q_2)$ and ~~f is not adjacent to y_1~~ f has a non-neighbour in Q_1 other than x_1.

2.2 Theorem 18.3, second conclusion

18.3 as printed (p. 110)

18.3 Let $G \in \mathcal{F}_7$, and let X, Y be disjoint nonempty anticonnected subsets of $V(G)$, complete to each other. Let P be a path $p_1\text{-}\cdots\text{-}p_n$ of $G \setminus (X \cup Y)$, where $n \geq 5$, such that p_1, p_n are the only X-complete vertices of P. Then P has even length. Assume that at least two vertices of P are Y-complete, and let P' be a maximal subpath of P such that none of its internal vertices are Y-complete. Then the length of P' has the same parity as the number of ends of P' that belong to $\{p_1, p_n\}$ and are not Y-complete. Moreover, the number of Y-complete edges of P has the same parity as the number of elements of $\{p_1, p_n\}$ that are Y-complete.

What is wrong. The second conclusion fails for subpaths of length 1. If $p_i p_{i+1}$ is a Y-complete edge of P with $1 < i$ and $i + 1 < n$, then $p_i p_{i+1}$ is a maximal subpath with no Y-complete internal vertex (it has no internal vertex, and neither one-step extension is admissible), of length 1, with zero ends in $\{p_1, p_n\}$: the printed claim reads $1 \equiv 0 \pmod 2$. Read literally the sentence forces P to have no Y-complete edge, which contradicts the third conclusion of 18.3 and the use in 18.4 ("P contains an odd number, at least 3, of Y-complete edges"). The paper's own proof carries the missing side condition: it opens "Let P' be a line of length ≥ 2", and its recap states the correct trichotomy, "a line has odd length if and only if *either it has length 1*, or one of its ends is one of p_1, p_n and is not Y-complete". The alternative was dropped when the recap became the statement.

The fix. The second conclusion is restricted to subpaths of length at least 2; the first and third conclusions are unchanged. The only consumer, 18.5, applies the conclusion to subpaths of length at least 3, so the paper's own uses are unaffected. The erratum is recorded in `formaliaxiv.json` and in the docstring of `Thm_18_3.lean`.

18.3 corrected

18.3 Let $G \in \mathcal{F}_7$, and let X, Y be disjoint nonempty anticonnected subsets of $V(G)$, complete to each other. Let P be a path $p_1\text{-}\cdots\text{-}p_n$ of $G \setminus (X \cup Y)$, where $n \geq 5$, such that p_1, p_n are the only X-complete vertices of P. Then P has even length. Assume that at least two vertices of P are Y-complete, and let P' be a maximal subpath of P such that none of its internal vertices are Y-complete, and suppose that P' has length at least 2. Then the length of P' has the same parity as the number of ends of P' that belong to $\{p_1, p_n\}$ and are not Y-complete. Moreover, the number of Y-complete edges of P has the same parity as the number of elements of $\{p_1, p_n\}$ that are Y-complete.

2.3 Theorem 2.5

2.5 as printed (p. 10)

2.5 Let G be Berge, let $X \subseteq V(G)$, and let P be a path in $G \setminus X$ of odd length, with vertices be $p_1\text{-}\cdots\text{-}p_n$, such that p_1, p_n are X-complete, and no edge of P is X-complete. Let $v \in V(G)$ be X-complete. Then either v is adjacent to one of p_1, p_2, or the only neighbour of v in P^* is p_{n-1}.

What is wrong. The hypothesis that X is anticonnected is missing. The neighbouring results 2.1, 2.2, 2.3 and 2.10 all carry it, and the proof of 2.5 opens "By 2.2, v has a neighbour in P^*" and continues "By 2.1, there is a leap a, b for P in X", both of which need X anticonnected. Without it the statement is false.

The fix. `thm_2_5` adds the hypothesis that X is anticonnected and nothing else. Its only use in the paper, in 2.8, has X anticonnected, so nothing downstream changes. The erratum is recorded in `formaliaxiv.json` and in the docstring of `Thm_2_5.lean`.

> **2.5 corrected**
>
> **2.5** Let G be Berge, let $X \subseteq V(G)$ be anticonnected, and let P be a path in $G \setminus X$ of odd length, with vertices $p_1\text{-}\cdots\text{-}p_n$, such that p_1, p_n are X-complete, and no edge of P is X-complete. Let $v \in V(G)$ be X-complete. Then either v is adjacent to one of p_1, p_2, or the only neighbour of v in P^* is p_{n-1}.

References

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A Minor issues: proof-level repairs

This appendix collects the changes that the system suggests to the printed text for clarity and completeness. None of them affects a theorem that the proof uses: the first two concern the summary material of Section 1 of the paper, and the rest concern individual sentences of printed proofs, which the Lean proofs repair in place. For each item we state what is printed, what is wrong or missing, and the suggested change.

Statement 1.8, item 4 (p. 7). Printed: "For every $G \in \mathcal{F}_1$, either G is an even prism with $|V(G)| = 9$, or G admits a proper 2-join or a balanced skew partition, or $G \in \mathcal{F}_4$." Since $\mathcal{F}_4 \subseteq \mathcal{F}_3$ and $\mathcal{F}_1 \not\subseteq \mathcal{F}_3$ the hypothesis cannot be $G \in \mathcal{F}_1$; an eight-vertex countermodel is $L(H)$ for $H = K_4$ with two opposite edges each subdivided once (Berge, in $\mathcal{F}_1$, not in $\mathcal{F}_3$, no even prism, no proper 2-join, no balanced skew partition). Fix: read $G \in \mathcal{F}_3$, which is what item 3 delivers, item 5 consumes, and 10.6 proves. Statement 1.8 is a summary of the proof and is not part of the Lean build, which proves 1.3 through 13.5.

The class $\mathcal{F}_5$ (p. 6). The class $\mathcal{F}_5$ is printed as "all $G \in \mathcal{F}_3$ such that no induced subgraph of G or of $\overline{G}$ is a long prism" (page 6), although the preamble says each class is a subclass of the previous one and $\mathcal{F}_4$ is the previous class. The Lean reproduces the print. Nothing is lost: an even prism has at least nine vertices and is therefore long, so "no long prism" already implies "no even prism" and $\mathcal{F}_5 \subseteq \mathcal{F}_4$ holds either way.

Proof-level repairs. Every numbered statement below is true and transcribed as printed; what fails is a sentence of the printed proof, and the suggested change is the repair the Lean proof carries out. Table 2 lists the sentences that are false as written, Table 3 the steps that omit a case, an unchecked hypothesis, or a construction. Page numbers are printed page numbers. Each row states the issue and the fix as it is carried out in the Lean.

Table 2: Printed proof sentences that are false as written.

Where (page)	Issue	Fix
2.11 (12)	"antipath Q with interior in $X \cup \{p_n\}$": $N(y) = X \cup \{p_1, p_n\}$ here.	Read with $X \cup \{p_1, p_n\}$; harmless.
4.1 (14)	"$N' = N \cup \{b_2\}$ is not anticonnected" fails when $N = \emptyset$.	Treat $N = \emptyset$ as a separate case.
4.2 (15)	"anticomponent of B' including $B \cup \{v\}$": $B \cup \{v\}$ is all of B'.	Read $B_1 \cup \{v\}$.
4.5 (17)	"we may assume $A_1 \subseteq L$, $B_1 \subseteq X$": the hypothesis needs $B_1 \subseteq Y$.	Use $B_1 \subseteq Y$.
5.7 (22–25)	G written for H three times; J used but never introduced.	Read as intended.
5.8 step (5) (27)	"holds by (5) or (2)" is a self-reference.	Read (4).
8.5 claim (2) (42)	"$X \subseteq \bigcup(S_{uv} : uv \in E(J))$": the full union always contains X.	Union over edges incident with v.
8.5 claim (4) (43)	"a unique pair (i, j)": ordered uniqueness fails at $n = 1$, and claim (6) then derives $n \geq 2$ from it circularly.	Edge-uniqueness; re-prove $n \geq 2$ from existence alone.
9.6 closing (55)	"$M_0 \neq \emptyset$, so G disconnected, and since $|V(G)| \geq 8$ it has a balanced skew partition": false for the edgeless graph.	Add the hypothesis that G has an edge, supplied by the striation.

Where (page)	Issue	Fix
10.6 closing (63)	"X = union of S_1 and all components attaching inside S_1": components with no attachment lie in all three X's.	Exclude them explicitly.
10.6 closing (63)	"$\|Y\| \geq 4$" rules out only the "length ≥ 3" half of the fourth 2-join bullet.	Use $\|A_2 \cup A_3\| \geq 2$.
11.4 (65)	"some vertex in B has a nonneighbour in F": F is anticomplete to B.	Read "in Q".
12.1 case (4) (70)	"otherwise it is minor, and statement 3 holds": a minor vertex gives outcome 1.	Read statement 1.
12.4 claim (4) (75)	"adjacent $a, b \in R_0^*$, both Q-complete" is not derivable from 2.1 when Q exceeds the antipath.	Replace by "complete to the interior $q_1 \dots q_k$"; the clause is never used.
16.1 claim (1) (98)	"since j odd, exactly one of $i-1$, $j-i$ is even": they always agree mod 2.	Use the flank lengths $i-1$, $j-i-1$.
16.1 claim (1) (98)	"from the symmetry we may assume $j \geq 4$": the minimality of Y' is not symmetric.	Case split on arc length, new argument for length 2.
18.4 (111)	"(C, Y) is an odd wheel": only one Y-complete edge is exhibited.	Obtain the second edge from the parity clause of 2.3.
18.6 claim (2) (114)	Parity count assumes P_1 has no Y-complete edge; fails when $b_1 = a_1 + 1$.	Argue via 2.10 (cap or leap).
18.6 claim (2) (114)	"either $b_2 = n$ or $a_2 = b_2 = 1$" presupposes $W_2 \neq \emptyset$.	Treat $W_2 = \emptyset$ separately.
19.2 claim (1) (118)	"By the minimality of $\|Y\|$ …" is a non sequitur: the induction gives a rim in $\{x_0, x_1, z\} \cup A_1$, and the printed minimality of A does not give y a neighbour in A.	Prove claim (1) with A_1; choose A minimal with the extra clause "y has a neighbour in A"; handle the new case $x_2 \sim y$ in claim (9).
19.2 claims (6), (10) (119–121)	"(C, Y_0) is a wheel" does not give every vertex of Y_0 a neighbour in $\{p_2, \dots, p_n\}$.	Use the edge count ≥ 2 from claim (2).
20.3 step (2) (125)	"the hole z-x_1-P-x_2-z" must be z-x_t-P-x_{t-1}-z.	Use the corrected hole.
21.2 claim (9) (137)	"contrary to $G \in \mathcal{F}_8$": the hypothesis is $G \in \mathcal{F}_7$ plus a no-pseudowheel clause.	Use the stated hypothesis.
23.2 (140)	"since G does not admit a skew partition": only "no balanced skew partition" is assumed.	Cite 15.1, as 23.4 does.
24.4 case 3 (144)	"by (1), n is odd, so $n \geq 4$": claim (1) gives even.	Use even.
24.7 (146)	"antipath of length ≥ 5, contrary to 24.5": length is ≥ 4 and 24.5 does not apply.	Use 24.6's "no antipath of length 4".
24.7 last sentence (146)	The cycle named is not a hole (fx_2 is a chord) and x_1 lies on it.	Use the hole x_2-f-P-f'-x_1-x_2 with external vertex x_1'.
24.7 other branch (146)	"hole of length ≥ 6": only ≥ 5 is known.	Apply 24.6, which needs only ≥ 5.

Table 3: Printed proof steps that omit a case, a hypothesis check, or a construction.

Where (page)	Issue	Fix
2.4 (10)	Case $v \in V(P_i)$ of the linking definition not treated.	Add the case.
2.8 (11)	Uses 2.2 and 2.5, which need $P_i \cap X = \emptyset$; not among 2.8's hypotheses.	Derive it.
2.9 (12)	Last sentence of the statement is never argued.	Prove it.
5.3 (20)	"a subgraph of H which is a subdivision of K_4" is Dirac's theorem, uncited, and must be applied to J, not H.	Prove Dirac; apply to J.
5.7 claim (1) (23)	"by Kőnig's theorem" is not Kőnig; cases $X = \emptyset$ and outcome 3 skipped.	Prove the needed lemma; add the cases.
5.7 claims (2), (4), endgame	Side conditions of every use of 5.6 unchecked; a track and a 2-cover asserted.	Seven gap lemmas.
5.8 singleton case (25)	5.7's "no even track of length ≥ 4" never verified.	Derive from Berge.
5.8 steps (3), (4) (26–27)	5.6's premises unchecked for the branch-deleted graph.	Check them.
5.8 step (6) (28)	The cycle C_2 through an A-edge and a B-edge is justified only by a parenthetical Menger sketch ("divide u into two adjacent vertices").	Carry the sketch out literally: split u, add a super-source and super-sink, apply Menger for two disjoint tracks.
5.8 coverage (28)	"(2)–(7) cover all possibilities": the branch/star case with ends of P exchanged is missing.	Weaken the helper to a disjunction.
5.4, 8.5 (2), 8.6 (1), §9	"there is an appearance of some J-enlargement with $L(H)$ induced" asserted three times, never proved.	Fourteen modules `EnlargementFromNonlocal*`.
6.1 claims (2), (7), (12) (31–33)	Existence of the third edge; the enlargement in $\overline{G}$; the length-1 case; a symmetry.	Supplied explicitly.
7.5 claim (2) (37)	Bipartiteness of H' after rung replacement is not derivable in the 5.8.2(a) case.	Prove parity from Berge via a return track.
8.1, 8.3, 8.6 (40–45)	Four unproved assertions (a cycle through uv; a mixed rung; $F' \neq \emptyset$; $H_0 \neq K_{3,3}$).	Prove each.
9.5 claim (2) (53)	Minimality of F transferred with no argument.	Close outcomes 9.3.2, 9.3.3 explicitly.
12.2 endgame (74)	"add p_1 to A" needs a rung through the new vertices.	Build it.
12.5 (77)	Fourth endpoint combination never mentioned.	By left–right symmetry.
13.7 (86)	Only one of 2.9's two bad alternatives refuted.	Mirror argument.
p. 86	"every recalcitrant graph is in $\mathcal{F}_5$, by 10.6 and 9.7" also needs 13.4, the 9-vertex clause of 10.6, and complement-invariance of homogeneous pairs.	Supplied.

Where (page)	Issue	Fix
16.1–16.3 (98–101)	Y' minimal in the wrong family; three translations between 16.1, 16.2, 16.3 unsaid.	Minimise where v is not complete; supply the translations.
16.3, 18.7, 23.2 “in particular”	Each theorem is one-sided while the class is two-sided.	Run on G and $\overline{G}$.
17.5 (106)	Case $X = \emptyset$; 17.2 cited where 17.3 is used.	Add the case; cite 17.3.
18.5, 18.7 (112–115)	Two appeals to 17.5 need only 17.4; a straddling argument after 18.6 unsaid.	Supplied.
19.2 claim (2) (118)	Hides a localisation to an induced subgraph and a parity step via 2.3.	Prove both (about 400 lines).
20.1, 20.5, 21.1–21.3 (123–137)	A hub clause unchecked; “$r \in A_{t-3}$” unargued; “length ≥ 6” only ≥ 5; “immediate” is not; half a hypothesis omitted.	Supplied.
22.1, 22.4, 22.2/22.5 (137–141)	A second X_t-complete vertex unsourced; $t \geq 1$ used with $t \geq 0$ given; a tail clause missing.	Supplied.
23.2 claim (5), 23.4 (141–142)	Only the first outcome of 2.11 gives the claim; on-hole vertices unsaid.	Handle the second outcome; add the trivial case.
24.4–24.6 (144–145)	The three-case trichotomy is a real theorem; “choose X maximal” for what is unsaid; “no antipath of length 4” is not trivial.	Prove the trichotomy; fix the admissible family; prove it.